BITS Pilani
K K Birla Goa Campus

Probability & Statistics,

Dr. Jajati Keshari Sahoo
Department of Mathematics

Continuous Distributions

- The Uniform Distribution
- Gamma Distribution
- Exponential Distribution
- Chi square Distribution
- Normal Distribution

Rectangular or Uniform distribution

A random variable *X* is said to have a continuous uniform distribution over an interval (***a***, ***b***) if its probability density function is constant *k* over entire range of *x*.

PROBABILITY DENSITY FUNCTION

$$f(x) = k, \qquad a < X < b$$

$$= 0 \qquad \text{otherwise}$$

Uniform distribution

Definition: A random variable *X* said to have uniform distribution over *(a, b)* if its density function is given by

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{for } a < x < b \\ 0 & \text{elsewhere} \end{cases}$$

Uniform distribution over any domain

Definition: A random variable X said to have uniform distribution over a finite domain D if its density function is given by

$$f(x) = \begin{cases} \dfrac{1}{\text{Length/Area/Volume of } D} & \text{for } x \in D \\ 0 & \text{elsewhere} \end{cases}$$

Uniform distribution

Distribution function (or CDF) for uniform density function is

$$F(x) = \begin{cases} 0 & \text{for } x \le a \\ \dfrac{x-a}{b-a} & \text{for } a < x < b \\ 1 & \text{for } x \ge b \end{cases}$$

Uniform Distribution

Mean of uniform distribution:

$$\mu = \frac{a+b}{2}$$

Proof:

$$\mu = \int_a^b x \cdot \frac{1}{b-a} dx = \frac{1}{b-a} \left. \frac{x^2}{2} \right|_a^b = \frac{a+b}{2}$$

Uniform Distribution

Variance of uniform distribution:

$$\sigma^2 = \frac{1}{12}(b-a)^2$$

Proof:

$$\mu_2 = \int_a^b x^2 \frac{1}{b-a} dx = \frac{1}{b-a} \left. \frac{x^3}{3} \right|_a^b = \frac{a^2+ab+b^2}{3}$$

Hence

$$\sigma^2 = \mu_2 - \mu^2 = \frac{a^2+ab+b^2}{3} - \frac{(a+b)^2}{4} = \frac{(b-a)^2}{12}.$$

Uniform Distribution

Moment Generating function of uniform distribution:

$$M_X(t) = \frac{e^{bt} - e^{at}}{t(b-a)}$$

Proof:

$$M_X(t) = \int_a^b e^{tx} f(x)\,dx = \frac{1}{b-a}\int_a^b e^{tx}\,dx$$

$$= \frac{e^{bt} - e^{at}}{t(b-a)}.$$

PROBLEM 1

If Subway trains on a certain line runs every half hour between midnight and six in the morning. What is the probability that a man entering the station at a random time during this period will have to wait at least twenty minutes?.

PROBLEM 2

If a string of length 1 meter is cut into 2 pieces at a random point along its length, find the probability that the longer piece is at least twice the length of the shorter one.

PROBLEM 3

If the random variable Y is uniform distributed over $(0,5)$, find the prob. that the root of the function $g(x) = 4x^2 + 4yx + (y+2)$ are real.

PROBLEM 4

Busses arrive at a specified stop at 15 minutes intervals starting at 7AM. If a passenger arrives at random at the stop at a time that is uniformly distributed between 7 and 7.30AM, find the probability that he waits

(a) less than 5 minutes for a bus;

(b) more than 10 minutes for a bus

PROBLEM 5

Trains headed for destination A arrive at the train station at 15-minute intervals starting at 7 a.m., whereas trains headed for destination B arrive at 15-minute intervals starting at 7:05a.m.

(a) If a passenger arrives at the station at a time uniformly distributed between 7 a.m. and 8 a.m. and then gets on the first train that arrives, what proportion of the time does this passenger go to destination A?